about Mach 3, cone angle and bluntness ratio are equally important; and above Mach 3, bluntness is the relatively stronger parameter in establishing the base pressure level. The validity of the correlation is demonstrated by comparison with specific 9 and 10 deg cone data for sharp and 0.3 bluntness ratio noses which are given in Fig. 4. Data for other cone angles and bluntness ratios exhibit an equally good agreement with the prediction as would be expected based on Fig. 2; however, all of these data are not given in Fig. 4 for clarity.

Also, total drag measurements obtained in an aeroballistics range on sharp 10-deg semiangle cones during the course of the transition experiments reported in Ref. 4 were analyzed. A sting free, flight base pressure was extracted from these data by subtracting a calculated forebody drag (pressure and skin friction) from the measured total drag. This calculations can be made with great precision for the sharp slender cone in question since the location of transition was measured. The relatively precise free-flight base pressure results obtained in this manner are given in Fig. 4 and further confirm both the absence of sting effects and the validity of the correlation. Even at Mach numbers in excess of eight, base pressure predictions utilizing this technique are in good qualitative agreement with the available turbulent data for sharp or small bluntness cones (up to Mach ~ 20).

Summary

At supersonic speeds, the base drag of sharp or blunt conical bodies represents a significant fraction of the total configuration drag. Correlation parameters are presented which reduce fully turbulent cone base pressures to a single curve for a wide range of nose bluntness ratios and cone semiangles. The base pressure which is computed using this technique is within ±10% of a fully turbulent flight equivalent value or better.

Acknowledgments

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Technical Comments

Comment on "Flutter of Flat Finite Element Panels in a Supersonic Potential Flow"

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PROFESSOR T. Y. Yang of Purdue University has just published a new solution[1] to one of this writer's thesis problems. Since the thesis[2] was written at UCLA almost two decades ago, and since there has been no interest in the two-dimensional, potential flow, panel flutter problem since 1965, it seems appropriate to ask "What's new?"

The thesis was intended to resolve some unexpected differences between the traveling wave solution for an unsupported infinite panel by Miles[3] (the writer's advisor) and the standing wave solution for a periodically supported infinite panel by Hedgepeth, Budiansky, and Leonard[4] of the NACA Langley Research Center. The second part of the thesis considered a finite length panel with equally spaced supports as an approximation to the periodically supported infinite panel. Three configurations were analyzed: single-span panels with pinned and clamped ends, and a two-span panel continous over pinned supports. A finite element formulation of the flutter problem used structural and aerodynamic influence coefficients (AICs). This formulation was an extension of the matrix method of Duncan and Collar[5] for free vibration analysis to solve the flutter problem by adding the aerodynamic forces via AICs. This extension was published in 1956.[6] The thesis developed the AICs for the two-dimensional panels and these were published later in dimensionless form in a survey paper[7] on unsteady AICs.

The single-span solutions using influence coefficients were compared with solutions of Shen[8] and Nelson and Cunningham[9] by the Galerkin method. It was seen that Shen's two-mode solution had not converged and that Nelson and Cunningham did achieve convergence with four modes by comparison with the nine-degree-of-freedom influence coefficient solution, and the accuracy of the influence coefficient formulation was established. However, only limited computed results were obtained in the thesis. Stability boundaries were not obtained at the lower Mach numbers ($M = 1.3$ and $\sqrt{2}$) but were obtained at the higher Mach numbers ($M = 1.56$ and 2.0), and the stability boundaries obtained at the high Mach numbers for the two-span panel were not correct. The latter difficulty was pointed out in 1963 by Dowell[10] in his thesis at MIT using Piston Theory, and the two-span stability boundary was corrected in 1964.[11] The corrected results agreed closely with Dowell's results and demonstrated that Piston Theory was adequate for two-dimensional panels at Mach numbers as low as 1.56. In 1964, Lock and Farkas, using the influence coefficient method, set out to study the single-degree-of-freedom stability boundary at low Mach numbers. Their results were reported in 1965.[12] At that point in time, the two-dimensional, potential flow, panel flutter problem was well understood, and investigators turned their attention to the three-dimensional problems of plates and shells.

The foregoing historical account permits us to answer the question "What's new?". Reference 1 uses the stiffness representation of the structure where the earlier finite element studies used a flexibility representation. Today, the stiffness formulation is preferable; before 1965, when NASTRAN was just being launched with equal requirements for the stiffness and flexibility approaches, it did not seem to matter. Reference 1 then uses Archer's consistent mass matrix[13] where the earlier studies had used a diagonal mass matrix. Today, the consistent mass matrix is preferable, just as a coupled mass matrix was employed in Ref. 6. However, with

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nine degrees-of-freedom per span, the first four natural frequencies and vibration modes of the panels were found to have sufficient accuracy and no mass coupling was necessary. Finally, we come to the less conventional matter of AICs. Equation (11) of Ref. 1 [i.e., Eq. (21) of Nelson and Cunningham[9]] which relates the panel deflections to the pressures, led to three terms in the AICs in the thesis.[2] The first term in Eq. (11) of Ref. 1 gives the static pressure and the AICs involve numerical differentiation; this is the important term at high Mach nuumber. The numerical differentiation in Ref. 1 has only cubic accuracy, whereas Refs. 2 and 7 have quartic accuracy. The AICs for the second term are diagonal. The AICs for the third term require numerical integration, which is as routine a matter in the field of numerical analysis as is differentiation. However, Prof. Yang regards it as a source of difficulty and remarks, "It is perhaps, this difficulty that hinders the finite element workers from employing the exact linearized flow theory"†; and later, "The inclusion of the higher-order frequency terms causes much complexity‡ in the formulation of an aerodynamic matrix." Equation (16) of Ref. 1 bases the third term on the trapezoidal rule with only linear accuracy, whereas Refs. 2 and 7 used an integration formula with cubic accuracy; we also note that the third term is the most important term at low Mach numbers. The final determination of the AICs in Eq. (17) of Ref. 1 is an additional averaging of the pressures over the length of the finite elements which increases the accuracy of the numerical differentiation and integration. Although this averaging increases the accuracy of the AICs probably to be comparable to Refs. 2 and 7, it does so at some computational expense.§

So, what's new? In terms of accuracy of formulation, we see that Ref. 1 offers us about the same, but requires a larger computational effort. In terms of results, Ref. 1 does not address the low Mach number problem per se, except at the isolated Mach number $M=1.3$, and therefore shows the wrong trend for thickness requirement in its Figs. 5 and 8; the correct trend in thickness requirement at low Mach number was shown by Lock and Farkas (Ref. 12, Figs. 9 and 10), and a maximum occurs near $M=1.3$. Reference 1 also shows some effects of tensile stresses, but their stabilizing effects are of little interest; it is the effects of compressive stresses that are a practical concern. In short, Prof. Yang has contributed nothing to improve upon the state-of-the-art of 1965.

To date, over 500 papers on panel flutter have been published, including a number of surveys, two of which are contained in the *AGARD Manual on Aeroelasticity,* and one textbook devoted entirely to the aeroelastic problems of plates and shells. The bibliography of these surveys is listed as Refs. 14-20. Reference 21 is added since it is a recently completed computer program (ZYNAPF) for routine design analysis of arbitrary three-dimensional panels including the effect of the boundary layer.

It is hoped that this Comment and the associated references will be useful to researchers and designers concerned with the panel flutter problem, and will discourage any further renaissance of the pressing problems of fifteen years ago.

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†We should point out that Ref. 7 also discussed three-dimensional AICs for flat and cylindrical panels, but only in a preliminary manner.

‡It does: the numbers are complex rather than real!

§This expense would be prohibitive in three-dimensional applications where the potential theory is based on, say, the supersonic Mach-box method.

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Reply by Author to W. P. Rodden

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DR. RODDEN misread the first sentence of the abstract, which states that "A finite element formulation and *solution procedure* are developed for the flutter analysis of

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